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Absence of determinism in El Niño Southern Oscillation

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We perform two direct determinism tests on the El Niño Southern Oscillation index monthly average series. The results indicate that, for timescales over 1 month, the series does not exhibit determinism, an essential feature of chaos.

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El Niño Southern Oscillation (ENSO) is a climatic phenomenon of great economic importance worldwide, and of much academic interest because of the complex interaction between oceanic and atmospheric phenomena which gives rise to it: see Ref. [1] for a comprehensive account, and Ref. [2] and references therein for more recent reviews.

A variety of models have been developed to study this phenomenon, among them low-dimensional, time delayed, hybrid, intermediate, and general coupled. While these explain the sustained oscillations of ENSO, its irregularities are not so well understood. In Ref. [2], three contending hypotheses are discussed: deterministic chaos, weather noise (atmospheric variability at fast time scales, combined with a periodic coupled model), and changing background state (slow variation of important parameters, such as mean thermocline depth.)

In this Rapid Communication, we partly address the first hypothesis. For many years, the existence of climatic chaotic attractors has been discussed in the literature for time series spanning nine orders of magnitude in time, ranging from 1 day to several million years [3]. ENSO has attracted interest [4,5] as a candidate chaotic system; there are recent claims [6,7] that the Southern Oscillation index (SOI) series has the characteristics of low-dimensional deterministic dynamics. SOI is the index that measures the pressure difference between Tahiti and Darwin in the Pacific Ocean; it is useful to indicate when the ENSO phenomenon is under way.

While many instances of chaos in observed hydrological and atmospheric data have been reported, there has been severe criticism of the use of nonlinear dynamics techniques for these data sets; see Ref. [8] for a recent review. In the present work we apply two tests of determinism to the SOI data. We find that the series appears to be nondeterministic, and hence not chaotic.

This paper is organized as follows. We begin by discussing nonlinearity tests in time series, and review their application to the SOI series. Then, we present two tests of determinism in observed time series. We apply these to the SOI series. Our main result is that the SOI series presents little or no evidence of low-dimensional deterministic behavior. Finally, we discuss our results and their consequences.

The indiscriminate application of time series techniques (correlation integrals, calculation of dimensions and exponents, etc.; see Ref. [9] for a comprehensive review of methods), can lead to the improper identification of chaotic behavior; see Ref. [10] for a specific example. To help avoid this problem, tests have been devised to verify two necessary conditions for chaos: nonlinearity and determinism. If either fails, one can reject the hypothesis that the time series is chaotic.

The nonlinearity of a time series can be tested by producing surrogate time series (see Ref. [11] for a recent review), which preserve linear characteristics of the original time series, such as power spectrum and autocorrelation function. One postulates the hypothesis that the time series to be tested is produced by a linear, correlated process, and that the possible nonlinearity of the system arises from filtering or distortion during measurement. If this were the case, both the tested and surrogate time series should perform comparably under a test related to nonlinear dynamics. A common test is the application of a nonlinear prediction algorithm to the series, with the average prediction error serving as a discriminating statistic. If the prediction error of the tested series is comparable to those of the surrogates, one can infer that the linear dynamics hypothesis is correct. On the other hand, if the prediction error in the tested series is considerably lower than that for the surrogates, one can conclude the existence of nonlinear dynamics in the generation of the test series.

In the first study [7], 50 surrogate series of the SOI were generated by a standard method that preserves the above-mentioned linear characteristics. Then, the last 134 points of each series were predicted through nonlinear algorithms. The authors rejected the null hypothesis (a linear Gaussian process) at a confidence level of 95% based on their results.

The second study uses a more accurate, newer scheme [12] and more stringent criteria. The results appear in a recent review of the surrogate time series method [11]. Here, 99 surrogates of SOI were generated and the nonlinear prediction errors of both the SOI and surrogate series were calculated. The test finds that the original series has a higher prediction error than about one-fourth of the surrogates, which means that the null (linear) hypothesis cannot be rejected.

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The authors offer the following possible explanations for their result: (1) Perhaps the prediction error statistics is not the most appropriate one; (2) the null hypothesis may be true, and the underlying dynamics is indeed linear; (3) the existing SOI data (1860 to present) is too short to explore the range of dynamics of ENSO, which may appear linear and stochastic without necessarily being so.

We now describe two tests for determinism, the second necessary condition for chaos. The first test [13] is based on the characteristic of chaotic attractors that the direction of motion of trajectories is a function of position in phase space. In other words, determinism implies that points that start close in phase space do not diverge very much over sufficiently short times. For longer times, sensitive dependence to initial conditions starts to separate trajectories.

If one performs a time series reconstruction followed by a coarse-grained partition of the phase space into boxes, one can do the following procedure. Suppose that n trajectory vectors pass through a particular box. For each trajectory vector, define a unit trajectory vector in the same direction, but with unit length. The average of the n directional vectors is the average directional vector for the box. Now, calculate the average length L of the average vectors for all boxes with the same number of passes n. In a deterministic system, L will be close to one, independent of n. (L will be slightly lower than one if the embedding dimension has been underestimated.) In a random system, the average length of the average directional vectors decays as $L \approx n^{-1/2}$.

For the second method [14], one picks a suitable time lag. Then, one applies the following procedure for different embedding dimensions. Pick N random points in the series, called centers (30 are used in Ref. [14]). Evolve all centers and their k nearest neighbors in phase space one time step. Calculate all the translations v_j. Then, compute the average translation for each group of points consisting of a center and its nearest neighbors, $\langle v \rangle$. For each group of points, calculate the average translation error

$$e = \frac{1}{k+1} \sum_{j=0}^{k} \frac{||v_j - \langle v \rangle||^2}{||\langle v \rangle||^2}, \qquad (1)$$

where $||v||$ denotes the length of v. The quantity e measures the normalized spread of the displacements around each center, relative to the average displacement. One expects e to be small for a deterministic series (of order 10^{-2}), and large (one or greater) for a random series. The examples presented in Ref. [14] are consistent with these expectations.

The two tests just described were applied to the monthly average SOI data (1860–2001). For each test we considered all delay times between 1 and 12 months, which covers the range of previously estimated time delays [6,7]. For both tests, the results for delay time $\tau \geqslant 2$ months do not change much, so we only show $\tau = 1$, 2, 4, and 8 months.

Figure 1 shows the results of the Kaplan-Glass test applied in three dimensions, with a 35^3-box partition. For all τ greater than 2 months, the average trajectory vectors coincide quite closely with what we expect for a random series. The $\tau = 1$ points are poised almost halfway between the random curve and the deterministic curve ($L = 1$); however, a decreasing tendency can be observed.

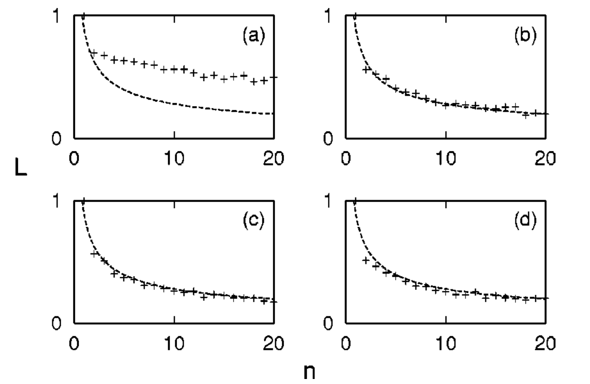

FIG. 1. Kaplan-Glass test for determinism of monthly SOI series, with 35^3 boxes. Length of average trajectory vector L vs number of passes n in a box. Purely deterministic series: $L \approx 1$; purely random series: $L = (4/\sqrt{6\pi}) n^{-1/2}$ (shown in dashed line). Time delays are (a) 1 month, (b) 2 months, (c) 4 months, and (d) 8 months.

Figure 2 shows the results for the Wayland test. We have used 200 centers. For all τ greater than 1 month, the average translation error is distinctly in the nondeterministic range. For 1 month, the error $0.5 \leqslant e \leqslant 2$ is still in the nondeterministic range, but with values somewhat lower, consistent with the results of the Kaplan-Glass test.

Based on these tests, we can conclude the following. The results for time delay of 1 month are ambiguous, and indicate at best a slight degree of dependence of the future on the past. This is reasonable, given that the SOI index conserves its sign for several months, for example, during ENSO events. This result is also consistent with the large (about 0.8) autocorrelation value of the SOI series with time delay of 1 month [7], and with the reasonably clean phase-space reconstruction obtain with this time delay [6].

For all delay times greater than 1 month, the SOI series fails both determinism tests; this indicates a severe breakdown of temporal causal correlations in the series after this

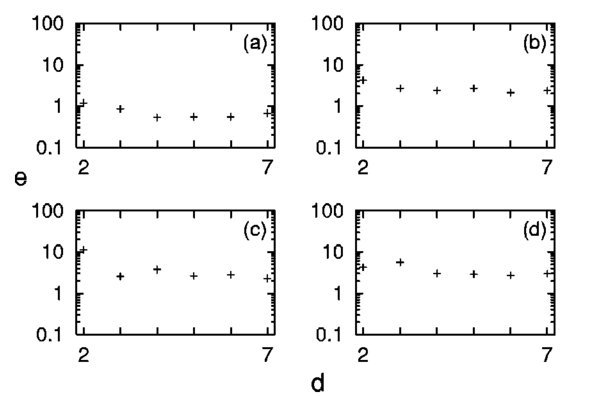

FIG. 2. Wayland *et al.* test of determinism for SOI series: average translation error e vs embedding dimensions $2 \leqslant d \leqslant 7$. Random series: $e \approx 1$; deterministic series: $e \approx 0.001 - 0.01$. Four time delays are shown: (a) 1 month, (b) 2 months, (c) 4 months, and (d) 8 months.

time scale. Our results indicate that the SOI series is not deterministic over the timescales of interest for prediction (of the order of a year or more). In contrast, the ENSO model developed by Vallis [4] shows considerable evidence of determinism. First, it is mathematically very similar to the Lorenz [15] system; both are systems of three coupled differential equations with their nonlinearity arising from terms that are products of two variables; both are of atmospheric or oceanic origin. The two-lobed structure of their attractors is almost identical. The Lorenz system was studied by Kaplan and Glass and by Wayland *et al.* [13,14], and passed both determinism tests. Second, other indirect yet useful signatures of determinism for the Vallis model, such as the mutual information and false nearest neighbors functions, have been found to be almost identical to those of the Lorenz system [16].

Our results, together with that of the most recent nonlinearity tests [11], strongly indicate that the SOI series does *not* exhibit low-dimensional chaotic behavior, at least up to the dimensions tested with the Wayland test. This is consistent with the fact that, to our knowledge, the SOI series itself has not been successfully used to predict ENSO events. It is, however, in apparent contradiction with the existence of deterministic equations that describe the ocean-atmosphere interaction, and with the continued improving efforts to predict the weather at least over short timescales. Therefore, some remarks are in order.

(1) Following Schreiber and Schmitz's conclusions on the surrogate data test, paraphrased in this paper, both nonlinearity and determinism tests are heuristic and can give incorrect results. Moreover, the available number of data points limits the dimension that can be tested.

(2) There are several reasons to expect the ENSO dynamics to be fairly high-dimensional. Among them, its spatiotemporal nature, the possible existence of time-delay mechanisms (see Ref. [5]) , and the fact that the governing equations may be nonautonomous (forced). While ENSO's coupling to the annual cycle has been established, its possible dependence on longer cycles, e.g., solar activity, perhaps should be studied. Moreover, Lorenz [17] has suggested that systems of a large number of variables with loose coupling can appear to be fairly low dimensional.

(3) The first of the above points (and the second, to some degree) suggests possible multiscale mechanisms in space or time. We may, then, view one of the three leading hypotheses, weather noise, as a high-dimensional deterministic mechanism that transfers fluctuations (mass, energy, or momentum) across scales.

(4) Finally, we report that we have obtained similar negative results when we apply determinism tests to other climatic data sets such as the Boston rainfall [18] and the Western Run stream flow in Maryland [8].

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